AP Calculus AB Summer Review Packet 2023

A solid understanding of algebraic and non-algebraic functions is necessary for our upcoming AP Calculus AB course. By completing the problem set below, you will be preparing to embark upon an exciting Calculus journey with me this fall. Although this packet should contain mostly review precalculus material, there might be a handful of problems or topics that are completely new to you. That is ok; do your best to figure it out.

- This problem set is due at the beginning of our first class.
- You may work with others though the goal of this review problem set is to solidify our precalculus foundation. Please keep that in mind if you are working with peers or tutors.
- Prepare your work and solutions on separate pieces of paper.
- Please complete all problems without a calculator. Show all work and leave answers in exact form such as $4\sqrt{2}$, $\frac{3\pi}{2}$, $6\ln 2$
- If you need help with a problem, try a search for the concept rather than the specific problem. For example, if you are not familiar with problem #6, try searching *difference quotient* to learn about the concept, then apply that concept yourself. Google, Khan Academy, or my YouTube channel might be helpful.
- You may not use answer machine apps such as Photomath or Mathway.
- This problem set was taken with permission by Mr. Yang from his website. You may consult his solutions to check your work.
- We will spend the first couple of classes reviewing precalculus, after which we will take a precalculus assessment.

Good luck and feel free to email or call me any time. Have a great summer and I will see you this fall.

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1) Find the x and y-intercepts for each of the following:

a) $y = x^2 + x - 1$

b) $y = (x-1)\sqrt{9-x^2}$

c) $y = \frac{x^2+3x}{(3x+1)^2}$

d) $x^2y - x^2 + 4y = 0$

2) Find all points of intersection of each of the following:

a) $2x - 3y = 13$ and $5x + 3y = 1$

b) $x^2 + y^2 = 25$ and $2x + y = 10$

c) $y = x^3 - 4x$ and $y = -x$

d) $y = x^4 - 2x^2 + 1$ and $y = 1 - x^2$

3) Write the equation of the line with the following characteristics

a) passes through (3, -4) and (5, 2)

b) is a horizontal line with a y-intercept at -4

c) is a vertical line that passes through (7, -8)

d) has an x-intercept at 5 and a y-intercept at -3

e) is parallel to the line $3x + 4y = 7$, passes through the point (-6, 4) and is written in point-slope form

f) is perpendicular to the line $5x - 3y = 0$, passes through the point $\left(\frac{3}{4}, \frac{7}{8}\right)$ and is written in point-slope form

4) For the function $f(x) = x^2 - 4x$, find each of the following:

a) $f(4)$ b) $f\left(\frac{3}{2}\right)$ c) $f(q)$ d) $f(t+4)$

5) Find the value of $\frac{f(x)-f(3)}{x-3}$ for each of the following functions:

a) $f(x) = 3x + 7$ b) $f(x) = 3x^2 - 2x + 1$ c) $f(x) = \frac{6}{x}$

6) Find the value of $\frac{f(x+h)-f(x)}{h}$ for each of the following functions:

a) $f(x) = 3x + 7$ b) $f(x) = 3x^2 - 2x + 1$ c) $f(x) = \frac{6}{x}$

7) For the piecewise function $p(x) = \begin{cases} \sqrt{x+4}, x \le 5 \\ (x-5)^2, x > 5 \end{cases}$ find each of the following:

a) $p(-3)$ b) $p(0)$ c) $p(5)$ d) $p(10)$

8) Write the following as piecewise functions: a) $y = |5x + 2|$ b) $y = |3 - 4x| + 8$

9) If $f(x) = \frac{3}{x}$ and $g(x) = x^2 - 1$, find a) $f(g(x))$ and b) $g(f(x))$ and state the domain of each.

10) Determine the domain for each of the following:

a) $g(x) = \frac{3x^2+2x-8}{2x^2+x-6}$ b) $h(x) = \sqrt{x} + \sqrt{1-x}$ c) $k(x) = \sqrt{x^2 - 3x + 2}$

11) Find the inverse of each of the following:

a) $f(x) = 4x - 3$ b) $g(x) = \frac{2x+3}{x-4}$ c) $h(x) = x^3 + 1$

12) For each of the following, find $f(-x)$ and use it to determine if the function is odd, even, or neither:

a) $f(x) = x^2(4 - x^2)$ b) $f(x) = \sqrt[3]{x}$ c) $f(x) = x \cos x$

13) Graph each of the following:

a) $f(x) = -3x + 2$ b) $f(x) = 1 - x^2$ c) $f(x) = \frac{1}{x}$ d) $f(x) = \begin{cases} 2x - 3, & x \le 3 \\ (x-6)^2 - 2, x > 3 \end{cases}$

e) $f(x) = |\,6 - x\,|$ f) $f(x) = \sqrt{9 - x^2}$ g) $f(x) = \ln(x - 1)$ h) $f(x) = e^x + 1$

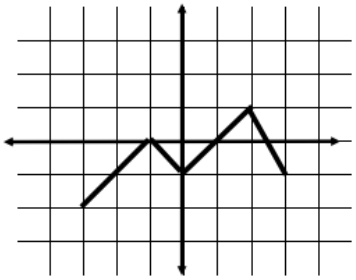
14) a – h) State the domain and range of each function graphed in problem #13

15) a – h) Use end behavior to identify $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for each function graphed in problem #13, with the exception of part g where you should find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to a^+} f(x)$ where a is the vertical asymptote for the function. You may skip part f for this problem.

16) a – h) State the intervals of x on which each function graphed in problem #13 is increasing or decreasing.

17) If the graph of $f(x)$ is shown below, graph each of the following:

a) $f(-x)$ b) $-f(x)$ c) $|f(x)|$ d) $f(x)+2$ e) $f(x+2)$



18) State the value of all 6 trig functions for each of the following angles:

a) $\frac{5\pi}{4}$ b) $\frac{11\pi}{6}$ c) $\frac{14\pi}{3}$ d) $\frac{\pi}{2}$ e) π

19) State the amplitude, period, phase shift and vertical shift for each of the following:

a) $y = 1 + \cos\left[3\left(x - \frac{\pi}{3}\right)\right]$ b) $y = 2 - 3\sin(4x + \pi)$

20) Simplify the following:

a) $\sin x \cos x \tan x \sec x \csc x$ b) $\frac{\sin x}{1+\cos x} + \frac{\sin x}{1-\cos x}$ c) $\cos^4 x + 2\cos^2 x \sin^2 x + \sin^4 x$

21) Solve the following (use identities where necessary) given that $0 \le x < 2\pi$

a) $\cos^2 x - \cos x + 1 = \sin^2 x$ b) $\sin x \tan x = \sin x$ c) $\sin x = \cos 2x - 1$

d) $\sin 4x = \frac{1}{2}$ e) $\cot^2 x - \csc x = 1$ f) $\sec^2 x + 2\sec x = 0$

g) $\sin x = \cos x$ h) $\sin 2x = \cos x$ i) $\cot x \cos^2 x = 2\cot x$

22) Simplify the following given that all angles are between 0 and π:

a) $\sin\left(2\arccos\frac{\sqrt{2}}{2}\right)$ b) $\cos\left(\arccos 0 + \arcsin\frac{1}{2}\right)$

23) Rewrite each of the following as an algebraic expression with no trig functions involved. (Hint: draw triangles and use Pythagorean Theorem.)

a) $\sin(\arccos 2x)$ b) $\cot(\arcsin x)$ c) $\sin(\arctan 3x)$

24) State horizontal asymptote(s), vertical asymptote(s) and hole(s) for each of the following:

a) $y = \frac{2x^2-7x-4}{6x^2+7x+2}$ b) $y = \frac{5x^2+20x}{x^3-3x^2-28x}$

25) Solve the following: a) $\frac{2x^2-7x-4}{6x^2+7x+2}=0$ b) $\frac{5x^2+20x}{x^3-3x^2-28x} = 0$

c) $\frac{2x^2-7x-4}{6x^2+7x+2} < 0$ d) $\frac{5x^2+20x}{x^3-3x^2-28x} \geq 0$ e) $\sqrt{3x+1} \leq 4$ f) $\sqrt{2x^2-13x+6} > 0$

26) Solve the following:

a) $\log_{81}\sqrt{3} = x$ b) $\log_x 64^{\frac{1}{3}} = \frac{1}{2}$ c) $9 = 4 + \log_2(x+3)$ d) $\frac{1}{3}\ln x = \ln 8$

e) $\log_b 8 = \log_b x + \log_b(x-2)$ f) $4\ln(x+3) = 12$ g) $e^{3x} = 6$ h) $7^{(x-4)} = 100$

27) Write each of the following using sigma notation:

a) 4 + 8 + 12 + 16 + 20 b) 1 + 4 + 9 + 16 + 25 + 36 + 49 c) 7 + 4 + 1 + -2 + ...

28) "BA" (BAD ALGEBRA) SECTION – The solution to each of the following equations contains at least one step (and possibly more) that involves bad algebra. Your job is to find the bad algebra, explain (very briefly) why it is bad algebra, and re-solve the problem correctly. (All BA's in ap calculus result in full credit lost for any problem, every time they occur.)

a) $10x^2 + 7x = 12$
$x(10x + 7) = 12$
$x = 12$ and $10x + 7 = 12$
$x = 12, x = \frac{1}{2}$

b) $(x - 5)^2 = 16$
$x^2 - 25 = 16$
$x^2 = 41$
$x = \pm\sqrt{41}$

c) $x^2 > 9$
$x > \pm 3$

d) $\sin 2x = \sin x, 0 \le x < 2\pi$
$2 \sin x = \sin x$
$\sin x = 0$
$x = 0$

e) $x^3 = x^2$
divide both sides by x^2
$x = 1$

f) $e^{3 \ln x} = 27$
$3x = 27$
$x = 9$

g) $\ln(x - 3) = 2$
$\ln x - \ln 3 = 2$
$\ln x = 2 + \ln 3$
$x = e^{2+\ln 3}$
$x = e^2 \cdot e^{\ln 3}$
$x = 3e^2$

h) $5x + 2x^{-1} = -11$
$\frac{5x+2}{x} = -11$
$5x + 2 = -11x$
$16x = -2$
$x = -\frac{1}{8}$

i) $2x^{-1} = 4$
$\frac{1}{2x} = 4$
$8x = 1$
$x = \frac{1}{8}$

j) $2(x - 1)^2 - x^2 = 14$
$(2x - 2)^2 - x^2 = 14$
$4x^2 - 4 - x^2 = 14$
$3x^2 = 18$
$x^2 = 6$
$x = \sqrt{6}$

k) $\sqrt{x^2 - 16} = 3$
$x - 4 = 3$
$x = 7$

l) $\frac{1}{x} + \frac{1}{2} = \frac{1}{3}$
$x + 2 = 3$
$x = 1$

m) $x = \sqrt{4}$
$x = \pm 2$

n) $\sqrt{x} = 9$
$x = 3$

BONUS (5 points): Find the coordinates of all points, *P*, <u>on the *x*-axis</u> so that the line through *A*(-3, -2) and *P* is perpendicular to the line through *B*(2, 7) and *P*.